

Models for Teaching Addition and Subtraction of Integers

The following are some everyday events that can be used to help students develop a conceptual understanding of addition and subtraction of integers.

Getting rid of a negative is a positive. For example:

Johnny used to cheat, fight and swear. Then he stopped cheating and fighting. Now he only has 1 negative trait so...

$$(3 \text{ negative traits}) - (2 \text{ negative traits}) = (1 \text{ negative trait}) \text{ or } (-3) - (-2) = (-1)$$

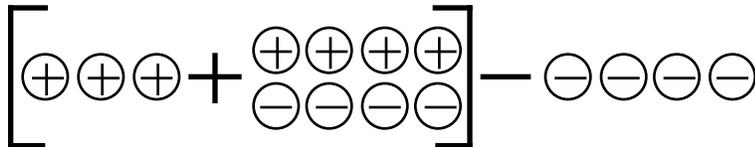
Using a credit card example can make this subtraction concept clearer. If you have spent money you don't have (-5) and paid off part of it (3), you still have a negative balance (-2) as a debt, or $-5 + 3 = -2$.

Draw a picture of a mountain, the shore (sea level) and the bottom of the ocean. Label sea level as 0.

Any of the following models can be used to help students understand the process of adding or subtracting integers. If students have trouble understanding and using one model you can show them how to use another model.

1. The Charged Particles Model

When using charged particles to subtract, $3 - (-4)$ for example, you begin with a picture of 3 positive particles, $\oplus \oplus \oplus$. Since there are no negatives in the beginning and you need them there to "take away" four negatives, $\ominus \ominus \ominus \ominus$, you introduce 4 pairs of positive and negative particles that are equivalent to 4 zeros.



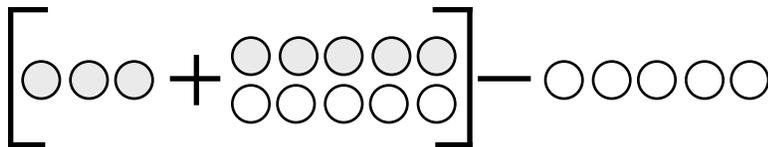
Now, take four of the negatives away from the zeros that you just introduced, and you are left with the original 3 positives and the four positives from the remaining part of the zeros which are combined (added) to get 7 as your solution.

$$\oplus \oplus \oplus + \oplus \oplus \oplus \oplus = 7$$

This is a great way to teach it, because it really shows why $3 - (-4) = 3 + 4$.

Two-color counters with one side (yellow) as positive and the other side (red) as negative can also be used. Accountants talk of "being in the red" when they have a negative cash flow. The rest of the modelling is similar to the Charged Particle model.

$$-3 - 5 =$$

$$\text{○○○} + \text{○○○○○○} = 8$$

2. The Stack or Row Model

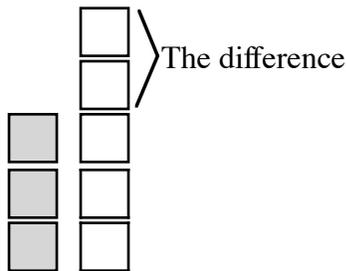
Use colored linking cubes and graph paper. Use graph paper and colored pencils for recording problems and results. Students should also write the problems in standard form and show the results that way.

Create stacks or rows of numbers with the colored linking cubes and combine/compare them. If the numbers have the same sign, then the cubes are the same color, and you *combine* them to make a stack or row. Thus $-3 + -4 = -7$ (all the same color).

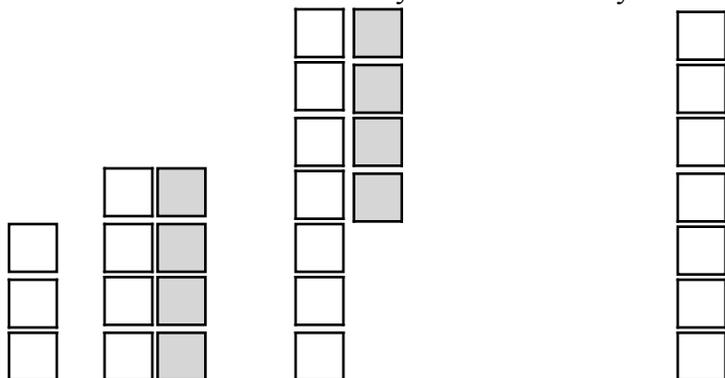
$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \square \\ \hline \end{array} + \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \square \\ \hline \square \\ \hline \end{array} = -7$$

This is also the model for $3 + 4 = 7$. This helps students understand the addition of integers with the same sign.

If the numbers are not the same sign (color), for example $-3 + 5 = 2$, you *compare* the stacks of different colors and the tallest stack wins. The result is the amount of difference between the stacks, easy to see and understand.



For **subtraction** you create zeros by pairing one of each color. Then add as many zeros to the first number as needed so that you can take away what the problem calls for. $3 - (-4)$

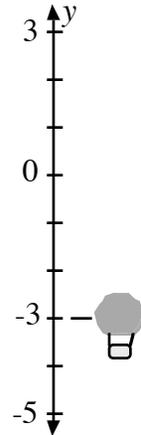


Now physically take away the indicated amount and see what is left.

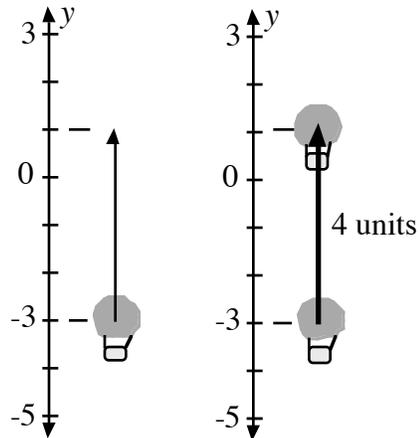
3. The Hot Air Balloon Model

Sand bags (negative integers) and *Hot Air bags* (positive integers) can be used to illustrate operations with integers. Bags can be put on (added to) the balloon or taken off (subtracted). Here is an example: $-3 - (-4) = ?$

The balloon starts at -3 (think of the balloon being 3 feet below sea level or 3 feet below the level of a canyon) and you take off 4 sand bags.

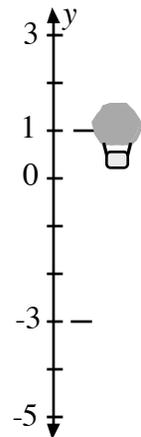


Now, think about what happens to a balloon if you remove sand bags. The balloon gets lighter. So, the balloon would go up 4 units.



If you think in terms of a *vertical number line*, it would start at -3 and end up at 1 , so $-3 - (-4) = 1$.

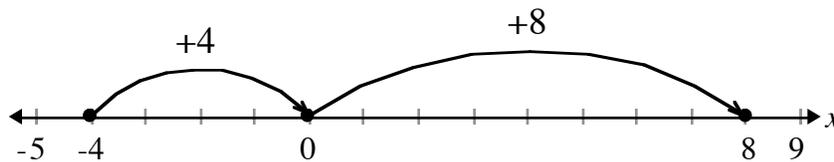
In order to have students make the connection between $-3 - (-4)$ and $-3 + (+4)$, present the addition and subtraction questions using the same numbers.



For example, the first addition question might be $9 + (-5)$ and the first subtraction question would then be $9 - (+5)$. The students see that putting on 5 *sand bags* produces the same result as taking off 5 *hot air bags*.

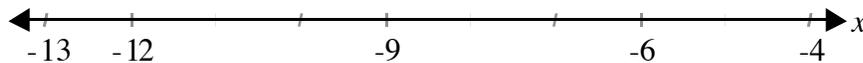
4. The Number Line Model.

You can describe subtraction of integers $a - b$ as the directional distance between b and a . Picture a number line and $8 - (-4)$.



The a represents the 8 and the b is (-4) . If we think about the directional distance from b to a we start at (-4) and move to the right until we reach 8. That move is $+12$ places. So $8 - (-4) = +12$.

Picture a number line and $-12 - -5 = -7$



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5. Postman Stories

A postman only brings financial mail.

- Sometimes she brings bad news, e.g., a bill for \$5 = -5 .
- Sometimes she brings good news - a check for \$5 = $+5$.

If she brings both you get two pieces of paper but zero dollars.

You always start a zero with a cash drawer full of matching checks and bills that equal zero dollars.

So if she brings me two checks for \$5, no sweat, she helps me by \$10, answer = $+10$. Similarly if she brings me 2 checks for \$5 the result is $2 \cdot +5 = +10$.

Now here is the tricky part: $-2 \cdot +5 = ?$ Well the $-$ sign means takes away from me. But if we start at zero how can she take anything away? This is where the cash drawer of matching checks and bills saves us. We just take away 2 checks and are left with 2 bills to pay. $-2 \cdot +5 = -10$.

Similarly, if she takes away our bills, she helps us and the money we would have used to pay the bill can now be spent on bubblegum. $-2 \cdot +5 = +10$.

The Postman Stories can be used to investigate signed numbers with students working in pairs. One student handles the bank (a ziploc bag filled with Monopoly money and paperclips). The other student holds a brown paper bag filled with checks and bills and delivers the mail.

The mailcarrier could begin by delivering two checks the first day (adding 2 positives). The mailcarrier's partner cashes in his/her checks at the bank (by placing the check in the ziploc bag and taking out the money). The next day, the mailcarrier might deliver two bills (adding 2 negatives) or a check and a bill (positive + a negative.). Either way, the student would pay a bill by attaching the monopoly money to the bill with a paperclip and keep it on the corner of his/her desk (sort of like in the mailbox). The student could also cash in a check by putting the check in the ziploc bag and taking out the money. Together, we record the transaction for each day, including how much was gained or lost that day.

After a few days, the mailcarrier might make a mistake (delivering the check or bill to the wrong house). If it was a check, that would be subtracting a positive. To get the check out of the bank, you would have to pay the bank (which would make you lose money). If it was a bill (taking away a negative), you can keep the money attached to the bill and give the bill back to the mailcarrier. This would show that taking away a negative would give you more money.

You can use similar situations for multiplication and division of signed numbers too! (Usually, you add a few of the same checks and a few of the same bills into the brown paper back to

multiply and divide.) To make it interesting, the fake checks are written out by teachers and administrators in the school. The bills are also school activity related.

You could use an envelope with an unknown amount of checks or bills in it to represent a variable. For example, $2x = -8$ would represent 2 envelopes (the same thing in each), which results in a loss of 8 dollars.